Counting Numbers

**COUNTING NUMBERS** are simply the numbers that we use for counting! They are also called **POSITIVE WHOLE NUMBERS.** Here is a list of the first 20 counting numbers:

**1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, …**

**Multiples**

**MULTIPLES** are the numbers that appear in your **TIMES TABLES**. Here are some examples:

**Multiples of 2:** { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, … }

**Multiples of 5:** { 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, … }

**Multiples of 12:** { 12, 24, 36, 48, 60, 72, 84, 96, 108, … }

**Multiples of 18:** { 18, 36, 54, 72, 90, 108, 126, 144, … }

**Multiples of 32:** { 32, 64, 96, 128, 160, 192, 224, … }

🖉 Here are some examples for you to copy and complete in your exercise book. Write down the first 6 multiples in each case. The first one has been completed for you as an example:

1. **Multiples of 3:** { 3, 6, 9, 12, 15, 18, … }
2. **Multiples of 7:**
3. **Multiples of 8:**
4. **Multiples of 10:**
5. **Multiples of 13:**
6. **Multiples of 16:**
7. **Multiples of 25:**

**Common Multiples and the LCM**

Look at these two lists of multiples:

**Multiples of 12:** { 12, 24, **36**, 48, 60, **72**, 84, 96, **108**, … }

**Multiples of 18:** { 18, **36**, 54, **72**, 90, **108**, 126, 144, … }

Some **multiples** are **common** to both lists. These are called the **COMMON MULTIPLES** of 12 and 18:

**Common Multiples of 12 & 18:** { 36, 72, 108, … }

The **lowest** number in this list of **common multiples** is 36, so we call this the **LOWEST COMMON MULTIPLE** of 12 and 18. We abbreviate this to **LCM** and write

**LCM(12,18) = 36** .

To find the LCM of two or more numbers, you follow this procedure:

1. List the **multiples** of each of the numbers;
2. Pick out the **common** multiples from the lists;
3. Select the **lowest** one of these and write this as the LCM.

For example, here is how we find the LCM of 3 and 5:

**Multiples of 3:** { 3, 6, 9, 12, **15**, 18, 21, 24, 27, **30**, 33, 36, 39, 42, **45**, 48, … }

**Multiples of 5:** { 5, 10, **15**, 20, 25, **30**, 35, 40, **45**, 50, 55, **60**, 65, 70, **75**, … }

**Common Multiples of 3 & 5:** { **15**, 30, 45, 60, 75, 90, 105, 120, 135, 150, … }

**LCM(3,5) = 15** .

🖉 Now answer these questions in your exercise book. Find the LCM of these numbers:

1. 2 & 7
2. 4 & 6
3. 2 & 13
4. 3, 4 & 6
5. 1, 2, 3 & 4

**Factors**

The **FACTORS** of a number are the numbers which will divide into that number exactly. For example, 1, 2, 3, 4, 6 and 12 are all factors of 12 because they all divide into 12 exactly, without leaving a remainder. But 5, 7, 8, 9, 10 and 11 are not factors of 12 because they all leave remainders when divided into 12. We write

**Factors of 12:** {1, 2, 3, 4, 6, 12} .

Notice that factors occur as **factor pairs** which multiply together to make 12:

1 x 12 = 12; 2 x 6 = 12; 3 x 4 = 12.

This can help you to check that you have listed all of the factors by linking the factor pairs like this:

**Factors of 12:** { 1, 2, 3, 4, 6, 12 }

Sometimes the middle factor is repeated as in this example:

**Factors of 9:** { 1, 3, 9 } and 1 x 9 = 9, 3 x 3 = 9 .

🖉 In your exercise book, list the factors of each of the numbers from 1 to 25. In each case, link the factor pairs with arrows (as in the previous two examples.)

**Unity, Prime and Composite Numbers**

🖉 Look at your lists of factors from the previous exercise and use them to help you to complete these sentences:

There is only one number having just **one factor**: the number is ……. ; this is called **UNITY**.

Those numbers that have exactly **two factors** are called **PRIME NUMBERS**; the prime numbers between 1 and 25 are ……………………………………………...…………………… .

All other numbers have **three or more factors**; they are called **COMPOSITE NUMBERS**; the composite numbers between 1 and 25 are ………………………………..………………... .

### Common Factors and the HCF

Here is a reminder of the **factors** of 16 and 24:

**Factors of 16:** { **1**, **2**, **4**, **8**, 16 }

**Factors of 24:** { **1**, **2**, 3, **4**, 6, **8**, 12, 24 }

Observe that there are some **factors** that are **common** to both lists. These are known as the **common factors** of 16 and 24. We write

**Common factors of 16 & 24:** { 1, 2, 4, **8** }

The **highest** value in this list is called the **highest common factor** of 16 and 24. We abbreviate this to **HCF** and write

#### HCF(16,24) = 8 .

To find the HCF of two or more numbers, you follow this procedure:

1. List the **factors** of each of the numbers;
2. Pick out the **common** factors from the lists;
3. Select the **highest** one of these and write this as the HCF.

For example, here is how we find the HCF of 18 and 30:

**Factors of 18:** { **1**, **2**, **3**, **6**, 9, 18}

**Factors of 30:** { **1**, **2**, **3**, 5, **6**, 10, 15, 30 }

**Common Factors of 18 & 30:** { 1, 2, 3, **6** }

**HCF(18,30) = 6** .

🖉 Now answer these questions in your exercise book. Find the HCF of these numbers:

1. 12 & 20
2. 14 & 15
3. 36 & 42
4. 8, 10 & 12
5. 18, 27 and 54

**EXTENSION WORK**

**Products, Prime Factors and Prime Decomposition**

First note that the **product** of the three numbers 3, 5 and 12, for example, is simply the number formed by **multiplying these numbers together**. So,

#### Product of 3, 5 and 12 = 3 x 5 x 12 = 180

(A number on its own, like 7 for example, is regarded as a **product** of one number, even though there is no multiplication sign!)

Next recall that the **factors** of 18 are as listed below:

**Factors of 18 =** { 1, **2**, **3**, 6, 9, 18 }

In this list of **factors**, 2 and 3 are **prime** numbers. So these are the **prime factors** of 18:

**Prime factors of 18 =** { 2, 3 }

Any counting number can be written as a **product** of its **prime factors**. This expression is called the **prime decomposition** of the counting number. So, in the above example,

#### Prime decomposition of 18 = 2 x 3 x 3 ,

or, using powers,

**Prime decomposition of 18 = 21 x 32** .

Here are some more examples of prime decompositions for you to study:

7 = 71, 24 = 23 x 31, 27 = 33, 126 = 21 x 32 x 71, 2600 = 23 x 52 x 131.

🖉 In your exercise book, work out the prime decompositions of each of these numbers. It is left to you to discover a suitable method!

1. 64, (2) 66, (3) 840, (4) 3125, (5) 221, (6) 2310, (7) 100, (8) 1000000.

Counting Numbers - Answers

**Multiples**

1. **Multiples of 3:** { 3, 6, 9, 12, 15, 18, … }
2. **Multiples of 7:**  { 7, 14, 21, 28, 35, 42, … }
3. **Multiples of 8:** { 8, 16, 24, 32, 40, 48, … }
4. **Multiples of 10:** { 10, 20, 30, 40, 50, 60, … }
5. **Multiples of 13:** { 13, 26, 39, 52, 65, 78, … }
6. **Multiples of 16:** { 16, 32, 48, 64, 80, 96, … }
7. **Multiples of 25:** { 25, 50, 75, 100, 125, 150, … }

**Common Multiples and the LCM**

1. **Multiples of 2:** { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, … }

**Multiples of 7:** { 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, … }

**Common Multiples of 2 & 7:** { 14, 28, 42, 56, 70, 84, 98, 112, … }

**LCM( 2, 7 ) = 14** .

2) **LCM( 4, 6 ) = 12** ; 3) **LCM( 2, 13 ) = 26** ;

4) **LCM( 3, 4, 6 ) = 12** ; 5) **LCM( 1, 2, 3, 4 ) = 12** .

**Factors & Unity, Primes and Composites**

**Factors of 1 =** { 1 } **Unity** **Factors of 2 =** { 1, 2 } **Prime**

**Factors of 3 =** { 1, 3 } **Prime** **Factors of 4 =** { 1, 2, 4 } **Composite**

**Factors of 5 =** { 1, 5 } **Prime** **Factors of 6 =** { 1, 2, 3, 6 } **Composite**

**Factors of 7 =** { 1, 7 } **Prime** **Factors of 8 =** { 1, 2, 4, 8 } **Composite**

**Factors of 9 =** { 1, 3, 9 } **Composite** **Factors of 10 =** { 1, 2, 5, 10 } **Composite**

**Factors of 11 =** { 1, 11 } **Prime** **Factors of 12 =** { 1, 2, 3, 4, 6, 12 } **Composite**

**Factors of 13 =** { 1, 13 } **Prime** **Factors of 14 =** { 1, 2, 7, 14 } **Composite**

**Factors of 15 =** { 1, 3, 5, 15 } **Composite Factors of 16 =** { 1, 2, 4, 8, 16 } **Composite**

**Factors of 17 =** { 1, 17 } **Prime Factors of 18 =** { 1, 2, 3, 6, 9, 18 } **Composite**

**Factors of 19 =** { 1, 19 } **Prime Factors of 20 =** { 1, 2, 4, 5, 10, 20 } **Composite**

**Factors of 21 =** { 1, 3, 7, 21 } **Composite Factors of 22 =** { 1, 2, 11, 22 } **Composite**

**Factors of 23 =** { 1, 23} **Prime Factors of 24 =** { 1, 2, 3, 4, 6, 8, 12, 24 } **Composite**

**Factors of 25 =** { 1, 3, 5, 15 } **Composite**

### Common Factors and the HCF

|  |  |  |  |
| --- | --- | --- | --- |
| 1) **Factors of 12:** { 1, 2, 3, 4, 6, 12 } | } |  |  |
|  | **Common factors of 12 & 20:** { 1, 2, 4 } | **HCF( 12, 20 ) = 4** . |
| **Factors of 20:** { 1, 2, 4, 5, 10, 20 } |  |  |

#### 2) HCF( 14, 15 ) = 1 ; 3) HCF( 36, 42 ) = 6 ; 4) HCF( 8, 10, 12 ) = 2 ; 5) HCF( 18, 27, 54 ) = 9 .

**EXTENSION WORK - Prime Decomposition**

1. **64 = 26**
2. **66 = 21 x 31 x 111**
3. **840 = 23 x 31 x 51 x 71**
4. **3125 = 55**
5. **221 = 131 x 171**
6. **2310 = 21 x 31 x 51 x 71 x 111**
7. **100 = 22 x 52**
8. **1000000 = 26 x 56**